New BRST Charges in RNS Superstring Theory and Deformed Pure Spinors

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Abstract

We show that new BRST charges in RNS superstring theory with nonstandard ghost numbers, constructed in our recent work, can be mapped to deformed pure spinor (PS) superstring theories, with the nilpotent pure spinor BRST charge $Q_{PS} = \oint \lambda^\alpha d_\alpha$ still retaining its form but with singular operator products between commuting spinor variables λ^α. Despite the OPE singularities, the pure spinor condition $\lambda\gamma^m\lambda = 0$ is still fulfilled in a weak sense, explained in the paper. The operator product singularities correspond to introducing interactions between the pure spinors. We conjecture that the leading singularity orders of the OPE between two interacting pure spinors is related to the ghost number of the corresponding BRST operator in RNS formalism. Namely, it is conjectured that the BRST operators of minimal superconformal ghost pictures $n > 0$ can be mapped to nilpotent BRST operators in the deformed pure spinor formalism with the OPE of two commuting spinors having a leading singularity order $\lambda(z)\lambda(w) \sim O(z - w)^{-2(n^2+6n+1)}$. The conjecture is checked explicitly for the first non-trivial case $n = 1$.

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Introduction

Pure spinor formalism [1], [2] is an efficient way of quantizing Green-Schwarz superstring theory in covariant gauge. It is also related to RNS description of superstring theory by mapping the pure spinor variables to those of RNS formalism (which can be performed with or without introducing non-minimal fields) [3], [4]. One advantage of the pure spinor formalism is the remarkably simple expression for the BRST operator:

$$Q_{PS} = \oint \frac{dz}{2i\pi} \lambda^\alpha d_\alpha$$
$$\alpha = 1, ..., 16$$
(1)

with the commuting spinor variable λ satisfying the pure spinor constraint [5], [6], [2]:

$$\lambda \gamma^m \lambda = 0$$
(2)

(where γ^m are the $d = 10$ gamma-matrices) and the action given by

$$S = \int d^2z \{ \frac{1}{2} \partial X_m \bar\partial X^m + p_\alpha \bar\partial \theta^\alpha + \bar p_\alpha \partial \bar\theta^\alpha + \lambda_\alpha \bar\partial w^\alpha + \bar\lambda_\alpha \partial \bar w^\alpha \}$$
(3)

where p_α is conjugate to θ_α [7] and w_α is bosonic ghost conjugate to λ^α. The action (3) is related to the standard GS action by substituting the constraint

$$d_\alpha = p_\alpha - \frac{1}{2}(\partial X^m + \frac{1}{4}\theta\gamma^m\partial\theta)(\gamma^m\theta)_\alpha = 0.$$
(4)

In our recent paper [4] we observed an isomorphism (up to similarity transformation) between BRST charges in pure spinor and RNS descriptions of superstring theory, that doesnt require non-minimal fields [4]. Namely, we observed that if one expresses the commuting spinor variable λ^α in terms of RNS fields (up to an overall normalization factor) as

$$\lambda_\alpha = be^{\frac{5}{2}\phi - 2\chi}\Sigma_\alpha + 2e^{\frac{3}{2}\phi - \chi}\gamma^m_{\alpha\beta}\partial X_m \tilde\Sigma^\beta - 2ce^{\frac{1}{2}\phi}\Sigma_\alpha \partial\phi - 4ce^{\frac{1}{2}\phi}\partial\Sigma_\alpha$$
(5)

satisfying $\lambda^\alpha = -4\{Q_0, \theta^\alpha\}$ with

$$\theta^\alpha = e^{\frac{1}{2}\phi}\Sigma^\alpha$$
(6)

being the RNS expression for the Green-Schwarz space-time fermionic coordinate and

$$Q_0 = \oint \frac{dz}{2i\pi} (cT - bc\partial c - \frac{1}{2}\gamma\psi_m \partial X^m - \frac{1}{4}b\gamma^2) \tag{7}$$

being the BRST charge in RNS theory, the pure spinor BRST charge (1) is related to RNS BRST charge (7) by the similarity transformation [4]. Here and elsewhere $\Sigma^\alpha(1, ..., 16)$ is the 16 component space-time spinor in RNS formalism, while $\tilde{\Sigma}^\alpha$ is used for the 16 component spinor with the opposite GSO parity.

Here ϕ and χ are the bosonized superconformal ghosts (appearing in the standard bosonization relations for the $\beta\gamma$-system [8]:

$$\gamma = e^{\phi - \chi}, \beta = e^{\chi - \phi}\partial\chi = \partial\xi e^{-\phi}$$

while the bc-system is bosonized in terms of a single free field σ:

$$b = e^{-\sigma}, c = e^{\sigma}.$$

The expression (6) for the θ^α variable is canonically conjugate to the space-time supercurrent $j^\alpha = e^{-\frac{1}{2}\phi}\tilde{\Sigma}^\alpha$ at picture $-\frac{1}{2}$. Alternatively, there also exists a picture $-\frac{1}{2}$ presentation for θ^α:

$$\theta^\alpha = ce^{\chi - \frac{3}{2}\phi}\Sigma^\alpha \tag{8}$$

which is the canonical conjugate to the space-time supercurrent at picture $\frac{1}{2}$ (note that, since θ^α is off-shell variable, picture-changing operation is only well-defined for the supercurrent (which worldsheet integral is on-shell) but not for θ^α itself; for this reason, the versions (6), (8) of θ^α are not directly related by the picture changing). We need both the version (6) and the version (8) of θ^α in order to mantain the picture uniformity in the d^α operator entering the expression for the pure spinor BRST charge, as d_α isn't uniform in θ. The RNS expression for the d_α variable in the pure spinor BRST charge, obtained from the expressions (6) and (8) for θ_α, is given by [4]

$$d_\alpha = e^{-\frac{1}{2}\phi}\tilde{\Sigma}_\alpha + 2c\xi e^{-\frac{3}{2}\phi}\gamma^m_{\alpha\beta}\partial X_m \Sigma^\beta - 32\partial c\partial\xi\xi e^{-\frac{5}{2}\phi}(\partial\tilde{\Sigma}_\alpha - \frac{19}{6}\tilde{\Sigma}_\alpha\partial\phi) \tag{9}$$

Using the RNS expression (5) for λ^α and the expression (9) for d_α, the straightforward computation of relevant OPEs was shown to map the pure spinor BRST charge (1) into the RNS BRST charge (7), up to the similarity transformation:

$$Q_0^{PS} \to e^{-R}Q_0^{RNS}e^R \tag{10}$$

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where

$$R = 16 \oint \frac{dz}{2i\pi} \partial cc \partial^2 \xi \xi e^{-2\phi} \partial \chi(z) \tag{11}$$

with

$$\partial^2 \xi \xi = 2e^{2\chi} \partial \chi \tag{12}$$

The pure spinor - RNS correspondence of the BRST charges (1), (7) contains, however, a subtle point. While the map (5), (9) formally relates nilpotent charges (1) and (7), the RNS expression (5) for λ^α is not literally a pure spinor variable of the free theory (3). That is, although the RNS expression (5) reproduces some properties of λ^α (it is a primary dimension 0 field and a commuting space-time spinor, given by BRST commutator with θ^α (6)), it only satisfies the pure spinor condition (2) in the weak sense, described below. That is, the OPE between two λ's is actually singular, with the leading order term being a double pole:

$$\lambda^\alpha(z)\lambda^\beta(w) \sim \frac{1}{(z-w)^2} \partial bb e^{5\phi-4\chi} \gamma^m_{\alpha\beta} \psi_m + \partial bb e^{5\phi-4\chi} \gamma^m_{\alpha\beta} (\frac{1}{4} \partial^2 \psi_m + \psi_m G^{(2)}(\phi, \chi, \sigma)) \tag{13}$$

where $G^{(2)}(\phi, \chi, \sigma)$ is a polynomial in the bosonized ghost fields of conformal dimension 2.

The appearance of the OPE terms on the right hand side does not, however, affect the nilpotence of the BRST charge (preserving the pure spinor - RNS correspondence (10)) since, as simple analysis shows, the normal ordered term of (13) has a vanishing normal product with with $\gamma^m \Pi_m$ where Π_m is defined by

$$d_\alpha(z)d_\beta(w) \sim -\frac{\gamma^m_{\alpha\beta}\Pi_m(w)}{z-w} + ..., \tag{14}$$

while the singular term in front of the double pole has vanishing normal ordering with the $(z-w)$ order term of the operator product $d_\alpha(z)d_\beta(w)$; for this reason no simple pole is produced in the OPE of the $\lambda^\alpha d_\alpha$ current with itself. So the weak pure spinor constraint

$$\lambda \gamma^m \lambda \approx 0 \tag{15}$$

is defined up to the terms with the vanishing normal ordering with the appropriate OPE terms of $d_\alpha(z)d_\beta(w)$.

Nevertheless, the appearance of the singular term in (13) indicates that although the commuting spinor variable (5) still satisfies the pure spinor condition (2) in the weak sense, it is not a pure spinor of a free theory (3) but of a theory with some interaction introduced.

That is, in order to produce the singularity in the OPE (13), the free action (3) has to be deformed with some interaction terms between the λ ghosts. Note that such a deformation would not generally affect the Green-Schwarz matter part of the action (3) (that appears as a result of imposing the constraint $d_\alpha = 0$) but only the ghost part.

The above observations imply that the pure spinor condition (2) for λ can be relaxed without violating the nilpotence of the BRST charge (1) : for instance, the nilpotence still would be preserved in a theory with interacting pure spinors (with singular operator products), if a pure spinor condition (2) is satisfied in the weak sense so that all the non-vanishing terms appearing in the OPE $lim_{z \to w} \gamma^m_{\alpha\beta} \lambda^\alpha(z) \lambda^\beta(w)$ have vanishing normal orderings with the appropriate terms of $d_\alpha(z) d_\beta(w)$. For example, consider the OPE between $d_\alpha(z) \ d_\beta(w)$ around the midpoint:

$$
\begin{aligned}
d_\alpha(z)d_\beta(w) = {} & -\frac{\gamma^m_{\alpha\beta}\Pi^{(1)}_m(\frac{z+w}{2})}{z-w} + (z-w)^0 \gamma^{m_1...m_3}_{\alpha\beta} \Pi^{(2)}_{m_1...m_3}(\frac{z+w}{2}) \\
& + (z-w)\{\alpha_1 \gamma^m_{\alpha\beta}\Pi^{(3)}_m + \alpha_2 \gamma^{m_1...m_5}\Pi^{(3)}_{m_1...m_5}\}(\frac{z+w}{2})
\end{aligned}
\tag{16}
$$

where α_1 and α_2 are some numbers and suppose that λ satisfies the OPE

$$
\lambda_\alpha(z)\lambda_\beta(w) \sim (z-w)^{-2}\gamma^m_{\alpha\beta}A_m(\frac{z+w}{2}) + \gamma^m_{\alpha\beta}B_m(\frac{z+w}{2})
\tag{17}
$$

Then the BRST charge is still nilpotent if : $B^m\Pi^{(1)}_m := 0$ and either $\alpha_1 = 0$ or : $A^m\Pi^{(3)}_m := 0$ (other singularities vanish upon evaluating traces of gamma-matrices). This precisely is the situation that is realised in case of the BRST charge of the form (1) in interacting pure spinor theory with quadratic OPE singularity (corresponding to Q_0 in RNS formalism) and it can be generalised for the interacting spinors with higher order OPE singularities (see below).

According to the calculation performed in [4], the theory of interacting pure spinors with the double pole OPE singularity can be mapped to standard BRST charge Q_0 of RNS theory, up to similarity transformation. It turns out [9] that in RNS theory one can construct, apart from Q_0 (related to superconformal symmetry of the theory), a sequence of nilpotent BRST charges Q_n, related to the ground ring of α-symmetries, found in our previous work. The construction is based on the local gauge symmetries on the worldsheet in RNS formalism, associated with the *global* space-time α-transformations, generating nonlinear space-time symmetries in RNS theory, mixing matter and ghost degrees of freedom, so that the variation of the matter part of the RNS action is cancelled by that of

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the ghost part. The generators of these symmetries can be classified in terms of the $b - c$ ghost cohomologies $R_{2n}(n > 0)$ (with n corresponding to the minimal *superconformal* ghost number of the "truncated" versions of these generators, inducing the s.c. *incomplete* space-time symmetries [9]). The expressions for the α generators typically depend on an arbitrary point w on the worldsheet ; however, this is a weak dependence since all the w derivatives of such operators are BRST exact. The BRST exact derivative operators, generate in turn the *local* worlsheet gauge symmetries in RNS theory. Identifying the symmetry algebra of these generators along with the associate generalized B and C-ghosts, it is straightforward to construct the related nilpotent BRST operators by the standard prescription [10]. The obtained BRST charges can be classified in terms of the ghost cohomologies H_n, as was explained in [9]. For example, the BRST charge of H_1, constructed in [9], is given by the integral of the current existing at minimal superconformal picture 1 (that is, it can't be related to any superconformal ghost pictures below 1) - unlike the usual BRST charge (7) that is given by the expression at the picture zero and can in principle be transformed to any other picture since it is an on-shell operator. Now the natural question to ask is: just as the standard BRST operator in RNS theory can be mapped to BRST operator in interacting pure spinor theory with the double pole (13) in the OPE between λ's, - are is there any interpretation of new nilpotent BRST operators Q_n in terms of theories with deformed pure spinors? In this paper we attempt to show that the answer to this question is positive - namely, that a sequence of nilpotent BRST charges found recently in RNS string theory corresponds to BRST charges of the form (1) of various deformed pure spinor theories , with the deformations preserving the nilpotence of the charge (1). More precisely, we shall argue that any RNS BRST charge in the cohomology class H_n (built on the gauge symmetries associated with the α-generators of $b - c$ cohomology R_{2n}) can be mapped to certain deformed pure spinor theory, so that the cohomology order n is related to the leading singularity order in the OPE of 2 λ's in the interacting pure spinor theory, given by $2n^2 + 12n + 2$. We shall demonstrate this map precisely for $n = 1$ (the first nontrivial case) and conjecture it for higher n's. The rest of the paper is organized as follows. In the section 2 we briefly review the basic properties of the local worldsheet gauge symmetries, associated with R_{2n} cohomologies and the related BRST charges. In the section 3 we show that the RNS BRST charge of minimal ghost number 1 (corresponding to the gauge symmetries associated with R_2) can be mapped to deformed pure spinor theory with the singular OPE of pure spinor variables $\lambda(z_1)$ and $\lambda(z_2)$ with the leading singularity order $(z_1 - z_2)^{-16}$. In the concluding section we attemt

to extrapolate our result in order to relate the sequence of BRST charges in RNS formalism to those in deformed pure spinor formalism with the leading orders of OPE singularities between pure spinors corresponding to the cohomology orders of new BRST operators in RNS formalism.

2. New BRST Operators in RNS Formalism

In this section we review the construction of a sequence of nilpotent BRST charges, found in our recent work [9], also adding some new observations. One starts with the "truncated" generators of global α-symmetries in space-time [11], [12], [9]. The "truncated" generators are typically not BRST invariant, generating "incomplete" versions of the space-time symmetries (incomplete in the sense that they do not involve the $b-c$ sector of the theory). On the contrary, the "full version" generators that are BRST-invariant and generate the complete space-time symmetry transformations, including the $b-c$ sector. The truncated generators are, however, useful as they can be naturally classified in terms of minimal superconformal ghost numbers they can have. Different ghost number versions of these generators can be related by direct or inverse picture changing transformations; however for an α-symmetry generator of minimal positive superconformal ghost number n there exist no ghost picture versions below n. The truncated generators can thus be divided into classes characterized by a minimal ghost number n. Structurally, the space-time symmetry generators characterized by minimal ghost number n have the form (if taken at minimal positive picture n) [9], [11],[12]:

$$L^{\alpha_n I} = \oint \frac{dz}{2i\pi} e^{n\phi} F^{\alpha_n I}_{\frac{1}{2}n^2+n+1}(z) \tag{18}$$

where $F^{\alpha_n I}_{\frac{1}{2}n^2+n+1}(z)$ are the matter primary fields of conformal dimension $\frac{1}{2}n^2 + n + 1$, while $\alpha_k(k = 1, ...n)$ and I are the indices labelling the generators (see below for explicit explanation for the indices). For example, in case of $n = 1$ for non-critical RNS superstring theory in d dimensions there are 2 generators $L^{\alpha_1 \pm}$ (so $I \equiv (\pm)$) with

$$F^{\alpha_1 +}_{\frac{5}{2}} \equiv F(X, \psi) = \psi_m \partial^2 X^m - 2\partial \psi_m \partial X^m$$

and

$$F^{\alpha_1 -}_{\frac{5}{2}} \equiv F(\varphi, \lambda) = \lambda \partial^2 \varphi - 2\partial \lambda \partial \varphi$$

where X^m, ψ^m are the space-time cooordinates and their worldsheet superpartners (RNS fermions) and φ, λ are components of the super Liouville field. Typically, in d space-time

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dimensions the generators of minimal ghost number n include 1 space-time d-vector and $n+1$ space-time scalars (so altogether there are $d+n+1$ space-time symmetry generators characterized by minimal ghost number n). As it has been shown that the symmetries induced by the generators (17) are closely related to hidden space-time dimensions, it is convenient to organize the indices α_k and I, labelling the generators, as follows. Namely, it has been shown [11], [12], [13], [9] that all the symmetry generators of the type (17) having minimal ghost numbers from 1 to N extend the full space-time symmetry group of d-dimensional RNS string theory (including the Liouville direction) from $SO(2,d)$ to $SO(2, d+N)$, increasing the number of space-time dimensions by N units. For each minimal ghost number $n(1 \leq n \leq N)$ the $d+n+1$ generators, characterized by n, increase the number of space-time dimensions by 1 unit (extending the symmetry group from $SO(2, d+n-1)$ to $SO(2, d+n)$), thus each minimal ghost number "contributes" a dimension. Labelling each induced space-time dimension with α_n (as before, n is a minimal ghost number of the associate symmetry generators), it is natural to label the $d+n+1$ generators (17) of minimal ghost number n as $L^{\alpha_n I}$ where the index $I = (m, \pm, \alpha_1, ... \alpha_{n-1})$ unifies d original space-time dimensions (labelled by $m = 0, ..., d-1$) and $n-1$ extra dimensions, induced by generators with minimal ghost numbers below n (labelled by $\alpha_1, ..., \alpha_{n-1}$). The $\pm$ indices are related to the Liouville direction (e.g. they distinguish between L^{m+} and L^{m-} generators that induce d translations in d-dimensional space time and d rotations in the Liouville-matter planes respectively).

As has been already pointed out, the space-time generators (17) are incomplete: they are not BRST-invariant (they don't commute with the supercurrent terms of Q_0) and generate truncated (incomplete) version of the α-symmetries. The $BRST$-invariant complete generators of α-symmetries can be obtained from the truncated generators (17) by using the K-operator procedure, defined as follows [9]: Let $L = \oint \frac{dz}{2i\pi} V(z)$ be some global symmetry generator, incomplete (in the sense described above) and not BRST-invariant, satisfying

$$[Q_{brst}, V(z)] = \partial U(z) + W(z) \tag{19}$$

and therefore

$$[Q_{brst}, L] = \oint \frac{dz}{2i\pi} W(z) \tag{20}$$

where V and W are some operators of conformal dimension 1 and U is some operator of dimension zero. Introduce the dimension 0 K-operator:

$$K(z) = -4ce^{2\chi - 2\phi}(z) \equiv \xi\Gamma^{-1}(z) \tag{21}$$

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satisfying

$$\{Q_{brst}, K\} = 1 \tag{22}$$

where $\xi = e^\chi$ and $\Gamma^{-1} = 4c\partial\xi e^{-2\phi}$ is the inverse picture-changing operator. Suppose that the K-operator (6) has a non-singular OPE with $W(z)$:

$$K(z_1)W(z_2) \sim (z_1 - z_2)^N Y(z_2) + O((z_1 - z_2)^{N+1}) \tag{23}$$

where $N \geq 0$ and Y is some operator of dimension $N + 1$. Then the complete BRST-invariant symmetry generator $\tilde{L}$ can be obtained from the incomplete non-invariant symmetry generator L by the following transformation:

$$\begin{aligned}
L \rightarrow \tilde{L}(w) = L + \frac{(-1)^N}{N!} \oint \frac{dz}{2i\pi}(z - w)^N : K\partial^N W : (z) \\
+ \frac{1}{N!} \oint \frac{dz}{2i\pi} \partial_z^{N+1}[(z - w)^N K(z)] K\{Q_{brst}, U\}
\end{aligned} \tag{24}$$

where w is some arbitrary point on the worldsheet. It is straightforward to check the invariance of $\tilde{L}$ by using some partial integration along with the relation (7) as well as the obvious identity

$$\{Q_{brst}, W(z)\} = -\partial(\{Q_{brst}, U(z)\}) \tag{25}$$

that follows directly from (4). The corrected invariant $\tilde{L}$-generators are then typically of the form

$$\tilde{L}(w) = \oint \frac{dz}{2i\pi}(z - w)^N \tilde{V}_{N+1}(z) \tag{26}$$

(see the rest of the paper for the concrete examples) with the conformal dimension $N + 1$ operator $\tilde{V}_{N+1}(z)$ in the integrand satisfying

$$[Q_{brst}, \tilde{V}_{N+1}(z)] = \partial^{N+1}\tilde{U}_0(z) \tag{27}$$

where $\tilde{U}_0$ is some operator of conformal dimension zero. Applying the K-transformation (23) to the symmetry generators (17) one finds that the BRST-invariant expressions for the α-symmetry generators (inducing the full version of the space-time symmetries) are given by:

$$\begin{aligned}
L^{\alpha_n I} \rightarrow \tilde{L}^{\alpha_n I}(w) = \frac{1}{(2n)!} \oint \frac{dz}{2i\pi}(z - w)^{2n}\{e^{n\phi}P_{2\phi-2\chi-\sigma}^{(2n)}F_{\frac{1}{2}n^2+n+1}^{\alpha_n I} \\
-4c\xi e^{(n-1)\phi}(\frac{1}{(n+1)!}P_{\phi-\chi}^{(n+1)}L_{\frac{1}{2}n^2+n+\frac{1}{2}}^{\alpha_n I} + \frac{f(n)}{n!}P_{\phi-\chi}^{(n+1)}\partial L_{\frac{1}{2}n^2+n+\frac{1}{2}}^{\alpha_n I} \\
+ \sum_{m=0}^{n-1} \frac{1}{m!(n-m-1)!}P_{\phi-\chi}^{(n-1-m)}\partial^m GF_{\frac{1}{2}n^2+n+1}^{\alpha_n I}) + g(n)\partial cc\partial\xi\xi e^{(n-2)\phi}F_{\frac{1}{2}n^2+n+1}^{\alpha_n I}\}
\end{aligned} \tag{28}$$

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Here $G = -\frac{1}{2}\psi_m\partial X^m$ is the matter part of the worldsheet supercurrent, $L^{\alpha_n I}_{\frac{1}{2}n^2+n+\frac{1}{2}}$ is the worldsheet superpartner of $F^{\alpha_n I}_{\frac{1}{2}n^2+n+1}$ satisfying the OPE

$$G(z)F^{\alpha_n I}_{\frac{1}{2}n^2+n+1}(w) = \frac{L^{\alpha_n I}_{\frac{1}{2}n^2+n+\frac{1}{2}}}{(z-w)^2} + \frac{f(n)\partial L^{\alpha_n I}_{\frac{1}{2}n^2+n+\frac{1}{2}}}{z-w} + ... \qquad (29)$$

and $f(n), g(n)$ are some numbers; for the $n = 1, 2, 3$ cases the value of $f(n)$ was computed to be equal to $\frac{1}{4}$, while $g(1) = 24, g(2) = 20, g(3) = 7$. For $n > 3$ cases the computation of the values of $f(n)$ and $g(n)$ is more complicated, requiring lengthy evaluations of cumbersome OPEs, although in principle it can be done explicitly. The complete invariant α-symmetry generators can be classified in terms of $b - c$ ghost cohomologies R_{2n} (where n refer to the minimal *superconformal* ghost numbers of the truncated symmetry generators prior to the K-transformation (23)) which are defined as follows (see also [9]). One starts with defining the notion of a $b - c$ picture for physical vertex operators, which is the generalization of a usual superconformal $\beta - \gamma$ picture. We define a physical operator $\tilde{L}(w)$ to have a $b - c$ picture n if it is represented in the form

$$L(w) = \oint \frac{dz}{2i\pi}(z-w)^n V_{n+1}(z)$$

where V_{n+1} is some operator of conformal dimension $n + 1$ satisfying

$$[Q_0, V_{n+1}] = \partial^{n+1} U_0 \qquad (30)$$

and U_0 is some operator of dimension 0. For example, the pictures -1 and 0 reproduce the familiar versions of the "unintegrated" and "integrated" vertex operators (e.g. a momentum zero photon operator is given by $c\partial X^m + \frac{1}{2}\gamma\psi^m$ at picture -1 and $\oint \partial X^m$ at picture zero). In addition, there are no nontrivial $b - c$ ghost pictures below -1. Just like the usual superconformal pictures can be raised by the transformation with the dimension zero picture-changing operator $\Gamma = \delta(\beta)\delta(S) \equiv: e^\phi S :$ (obtained by the integration over the fermionic supermoduli of gravitini in functional integrals for scattering amplitudes with S being the full matter+ ghost supercurrent), the $b - c$-pictures are raised by the dimension zero invariant Z-operator, that also can be obtained from functional integrals for RNS scattering amplitudes by integration over the *bosonic* moduli of the worldsheet metric [14]:

$$Z(w) = \delta(b)\delta(T) \equiv b\delta(T)(w) = \oint \frac{dz}{2i\pi}(z-w)^3(bT + 4c\partial\xi\xi e^{-2\phi}T^2)(z) \qquad (31)$$

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where T is the full matter+ghost stress-energy tensor. Just as the standard picture chang-ing operator can be written as the BRST commutator outside the small Hilbert space: $\Gamma = \{Q_0, \xi\}$, the Z-operator (30) is also given by the BRST commutator outside the small Hilbert space:

$$Z(w) = -[Q_0, \oint \frac{dz}{2i\pi}(z - w)^4 \partial\xi\xi e^{-2\phi}T^2(z)] \tag{32}$$

Typically, the action of Z on a physical vertex operators at $b - c$ ghost picture n is given by (after integrating out total derivatives):

$$Z(w)\tilde{L}_{[n]}(w) \equiv Z(w) \oint \frac{dz}{2i\pi}(z - w)^n V_{n+1}(z) = \tilde{L}_{[n+1]}(w) \equiv \oint \frac{dz}{2i\pi}(z - w)^{n+1} V_{n+2}(z) \tag{33}$$

where V_{n+1} and V_{n+2} both satisfy (29) (generally, with the different U_0's). For example, acting with Z on elementary vertex operators (such as photon) at $b - c$ picture -1 (known as "unintegrated" vertices) one obtains vertex operators at $b - c$ picture zero (known as "integrated" vertices) One can also define the Z^{-1} operator

$$Z^{-1} = [Q_0, \xi(\partial c - c\partial\phi)] \tag{34}$$

formally satisfying

$$\{Z^{-1}, Z\} =: \Gamma : +[Q_0, ...] \tag{35}$$

Having made all these definitions, we are now prepared to define the $b - c$ ghost cohomologies R_N. The definition is quite similar to that of the superconformal $(\beta - \gamma)$ ghost cohomologies described in [9] and other works. The $b - c$ ghost cohomology R_N consists of physical (BRST-invariant and nontrivial) vertex operators, *violating* the equivalence of the $b - c$ ghost pictures (defined above), that exist at *minimal* $b - c$ ghost picture $N > -1$ and *cannot* be related to $b - c$ pictures less than N by any Z-transformation; the Z-transformations can, however, relate it to $b - c$ pictures higher than N $(N + 1, N + 2, ...)$ so the elements of R_{2N} exist at pictures N and above, but not below N. As it is clear from the above definitions, the complete BRST-invariant α-symmetry generators $\tilde{L}^{\alpha_n I}$ obtained in (27) are the elements of R_{2n}. That is, the truncated non-invariant global symmetry generators $L^{\alpha_n I}$ (17) inducing incomplete symmetry transformations classified by the minimal *superconformal* ghost number n, become the elements of the $b - c$ ghost cohomology R_{2n} as a result of the K-transformation, defined in (23).

The peculiar property of the full symmetry generators (27) is that, while they induce global nonlinear symmetries in space-time, they also depend on an arbitrary point w on

the worldsheet, except for the trivial case $N = 0$. The $N = 0$ case is realised, for example, when the K-transformation (23) is applied to the incomplete (and BRST non-invariant) to the space-time rotation generator $L^{mn} = \oint \frac{dz}{2i\pi} \psi^m \psi^n$ which induces space-time rotational symmetries for RNS fermions, but not for RNS bosons; when the K-transformation is applied to L^{mn}, the obtained operator $\tilde{L}^{mn}$ - BRST-invariant and complete - generates the space-time rotation for the full set of the matter fields (up to picture-changing for X's)). The K-transformation applied to the truncated α-generators requires, however, $N = 2n$, hence the full invariant generators (27) appear to manifestly depend on w. This ambiguity, however, is resolved if we note that all the $2n$ non-vanishing derivatives in w of the complete α-generators $\partial^k \tilde{L}^{\alpha_n I}(w)(k = 1, ..., 2n)$ are BRST-exact:

$$\partial^k \tilde{L}^{\alpha_n I}(w) = \{Q_0, \partial^{k-1}(b_{-1}\partial^k \tilde{L}^{\alpha_n I}(w))\} \tag{36}$$

In particular, in the $k = 1$ case we have

$$\partial \tilde{L}^{\alpha_n I}(w) = \{Q_0, b_{-1}\tilde{L}^{\alpha_n I}(w)\}. \tag{37}$$

implying that the left hand side of (36) (expression of conformal dimension 1 with the integrand of conformal dimension 2) is the analogue of the worldsheet integral of the stress-energy tensor $\oint T$, given by the BRST commutator with the worldsheet integral of generalized b-ghost

$$\oint B^{\alpha_n I}(w) = b_{-1}\tilde{L}^{\alpha_n I}(w) \tag{38}$$

where b_{-1} is the worldsheet integral of the standard b-ghost field: $b_{-1} = \oint \frac{dz}{2i\pi} b(z)$. Next, remarkably, it can be shown [9] that the derivatives of the full *global* space-time α-symmetry generators $\tilde{L}^{\alpha_n I}(w)$ generate *local* gauge symmetries on the worldsheet - just like the stress-energy tensor and its derivatives, given by the anticommutator of the BRST charge with the b-ghost and its derivatives, generate the superconformal symmetries on the worldsheet. The difference, however, is that while the superconformal symmetry is infinite-dimensional (particularly generated by the stress tensor T, the worldsheet supercurrent and the infinite number of their derivatives), the gauge symmetries induced by the derivatives of the α-generators (27) are finite-dimensional, since the number of the non-vanishing derivatives is finite (equal to $2n$ for each $\tilde{L}^{\alpha_n I}$). Nevertherless, it turns out that the algebra of the gauge symmetries generated by the derivatives of $\tilde{L}^{\alpha_n I}(w)$ does have a conformal-like

structure, reminiscent of the "truncated" finite-dimensional Virasoro algebra [9]. Namely, defining

$$L_k^{\alpha_n I} = \partial^k \tilde{L}^{\alpha_n I} \tag{39}$$

and rescaling

$$T_k^{\alpha_n I} = \frac{L_k^{\alpha_n I}}{(n-k)!} \tag{40}$$

it can be checked explicitly (for $n = 1, 2, 3$ and extrapolated for higher values of n) that the properly normalized local gauge symmetry generators $T_k^{\alpha_n I}(k = 1, ..., 2n)$ satisfy the following commutation relations:

$$
\begin{aligned}
[T_{k_1}^{\alpha_{n_1} I}, T_{k_2}^{\alpha_{n_2} J}] = (k_1 - k_2)\{ & \theta(max(2n_1, 2n_2) - k_1 - k_2)\eta^{IJ} T_{k_1+k_2}^{\alpha_{n_1} \alpha_{n_2}} \\
& +\theta(max(2N(I), 2N(J)) - k_1 - k_2)\delta^{\alpha_{n_1} \alpha_{n_2}} T_{k_1+k_2}^{IJ} \\
& +\theta(max(2n_2, 2N(I)) - k_1 - k_2)\delta^{\alpha_{n_1} J} T_{k_1+k_2}^{\alpha_{n_2} I} \\
& -\theta(max(2n_1, 2N(J)) - k_1 - k_2)\delta^{\alpha_{n_2} I} T_{k_1+k_2}^{\alpha_{n_1} J} \}
\end{aligned}
\tag{41}
$$

where $\theta(n)$ is a usual step function (taken at discrete values of the argument), equal to 1 for $n \geq 0$ and 0 for $n < 0$; the function $max(m, n) = m$ if $m > n$ and $max(m, n) = n$ otherwise; finally, $2N(I)$ is the value of the cohomology order associated with the index I (i.e. $N(I) = k$ if $I = \alpha_k$ and $N(I) = 0$ if I stands for $+, -$ or a d-dimensional space-time index m). The step function factors in the commutators (40) ensure that for each of the terms on the right hand side of (40) the derivative order $k_1 + k_2$ of the underlying global α-symmetry generator $L^{\alpha_N I}$ is less or equal to its $b - c$ cohomology order $2N$; since all the higher order derivatives $L_k^{\alpha_N I} \equiv \partial^k \tilde{L}^{\alpha_N I}$ vanish identically if $k > 2N$ (so that each global α- generator of R_{2N} gives rise to $2N$ local gauge symmetry generators). So structurally the commutation relations (40) take the form

$$[L_m^M, L_n^N] = (m - n) f_{\underline{P}}^{\underline{MN}} L_{m+n}^{\underline{P}} \tag{42}$$

(where for convenience the underlined indices unify α_i and I in the gauge symmetry generators) provided that $m + n$ is less or equal to the R-cohomology order of $\tilde{L}^{\underline{P}}$. Here $f_{\underline{P}}^{\underline{MN}}$ are the structure constants inherited from the global symmetry algebra of the α-generators while the origin of the $(m - n)$ factor is Virasoro-type, related to the local worldsheet properties of the generators (38), (39).

Given the gauge symmetry generators (35), it is straightforward to check that the generalized B-ghost fields (37) $\oint B^{\alpha_n I} = b_{-1} \tilde{L}^{\alpha_n I}(w)$ are in the adjoint of the gauge

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symmetries (40). Thus the only components missing for the construction of nilpotent BRST charge are the generalized C-ghosts that must be canonical congugates of the B-ghosts. Thus in order to construct the C-ghosts one has to identify local primary fields of dimension -1 satisfying

$$\{\oint B^{\alpha_n I}, C^{\alpha_n I}\} = 1 \tag{43}$$

(in full analogy with the usual $b-c$ ghosts). Such objects have been constructed explicitly for the $n = 1, 2, 3$ cases. The explicit expressions are given by [9]:

$$C^{\alpha_1 I} =: e^\phi G e^{2\phi-\chi} L_2^{\alpha_1 I} : + [\tilde{Q}_0, be^{3\phi-\chi} F_{\frac{5}{2}}^{\alpha_1 I} P_{\phi-\chi-\frac{3}{4}\sigma}^{(1)}] \tag{44}$$

for the gauge symmetries derived from the α-generators of R_2,

$$C^{\alpha_2 I} = e^\phi G e^{3\phi-\chi} L_{\frac{9}{2}}^{\alpha_2 I} P_{\phi-\frac{10}{3}\chi}^{(1)} + \{\tilde{Q}_0, \partial^2 b\partial bbe^{5\phi-2\chi} L_{\frac{9}{2}}^{\alpha_2 I}\} \tag{45}$$

for the gauge symmetries derived from the α-generators of R_4 and

$$C^{\alpha_3 I} =: e^\phi G e^{4\phi-\chi} (P_{\phi-3\chi}^{(1)} \partial L_8^{\alpha_3 I \alpha_3 I} + \frac{4}{5} L_8^{\alpha_3 I} (\partial P_{\phi-3\chi}^{(1)} + P_{\phi-3\chi}^{(1)} P_{4\phi-\chi}^{(1)})) :$$
$$+ \{\tilde{Q}_0, \partial^4 b\partial^3 b\partial^2 b\partial bbe^{7\phi-3\chi} (\partial P_{\phi-\frac{9}{4}\chi}^{(1)} + P_{\phi-\frac{9}{4}\chi}^{(1)} P_{7\phi-3\chi-7\sigma}^{(1)}) F_{ij(\frac{17}{2})}^{matter}\} \tag{46}$$

for the gauge symmetries derived from the α-generators of R_6. Here $\tilde{Q}_0 = Q_0 - \oint \frac{dz}{2i\pi}(cT - bc\partial c)$ stands for the supercurrent part of the standard BRST charge and $G = -\frac{1}{2}\psi_m \partial X^m$ is the matter part of the worldsheet supercurrent. It should be noted that $C^{\alpha_1 I}, C^{\alpha_2 I}, C^{\alpha_3 I}$ ghost fields exist at the minimal superconformal picture 2,3 and 4 respectively; therefore in order to satisfy the canonical relations (42) the $\oint B^{\alpha_i I}$ ghost fields ($i = 1, 2, 3$) have to be picture transformed to superconformal pictures $-2, -3$ and -4 respectively by using the inverse picture changing. Note that, even though $\oint B^{\alpha_i I}$ are not on-shell operators, picture-changing transformations (both direct and inverse) are still well-defined for them since $L_1^{\alpha_i I} = \{Q_0, \oint B^{\alpha_i I}\} \equiv \{Q_0, b_{-1}\tilde{L}^{\alpha_i I}\}$ and $\tilde{L}^{\alpha_i I}$ is an on-shell operator. Unfortunately for $n > 3$ the expressions for the gauge symmetry generators are increasingly cumbersome and the manifest construction of the generalized C-ghosts becomes complicated. With the gauge symmetry generators (35), (39), the generalized B-ghosts (37) in the adjoint of the gauge symmetry group and the generalized C-ghosts (42) - (45) it is straightforward to construct the nilpotent BRST charges related to these gauge symmetries. By definition, the BRST charge is given by

$$Q = \sum_{n,\underline{N}} C_n^{\underline{N}} T_n^{\underline{N}} + \frac{1}{2} \sum_{m,n,\underline{M},\underline{N},\underline{P}} (m-n) f_{\underline{P}}^{\underline{MN}} C_m^{\underline{M}} C_n^{\underline{N}} B_{n+m}^{\underline{P}} \tag{47}$$

where $C_n^{\underline{N}} = \oint \frac{dz}{2i\pi} z^{n-1} C^{\underline{N}}$ and $B_n^{\underline{N}} = n! \oint \frac{dw}{2i\pi} \frac{1}{w^{n+1}} b_{-1} \tilde{L}^{\underline{N}}$

The manifest expression for BRST charge is particularly simple if we restrict ourselves to the operators of the first non-trivial cohomology R_2. In this case there are 2 commuting operators with identical structure $\tilde{L}^{\alpha_1\pm}$:

$$\tilde{L}^{\alpha+}(w) = \frac{1}{2} \oint \frac{dz}{2i\pi} (z-w)^2 [\frac{1}{2} e^{\phi} F(X,\psi) P^{(2)}_{2\phi-2\chi-\sigma}(z) +$$
$$4c\xi(F(X,\psi)G - \frac{1}{2}L(X,\psi)P^{(2)}_{\phi-\chi} - \frac{1}{4}\partial L(X,\psi)P^{(1)}_{\phi-\chi}) - 24\partial cc e^{2\chi-\phi}F(X,\psi)]$$

(48)

$$\tilde{L}^{\alpha-}(w) = \frac{1}{2} \oint \frac{dz}{2i\pi} (z-w)^2 [\frac{1}{2} e^{\phi} F(\varphi,\lambda) P^{(2)}_{2\phi-2\chi-\sigma}(z) +$$
$$4c\xi(F(\varphi,\lambda)G_L - \frac{1}{2}L(\varphi,\lambda)P^{(2)}_{\phi-\chi} - \frac{1}{4}\partial L(\varphi,\lambda)P^{(1)}_{\phi-\chi}) - 24\partial cc e^{2\chi-\phi}F(\varphi,\lambda)]$$

(49)

where, as previously,

$$F(X,\psi) \equiv F_{\frac{5}{2}} = \psi_m \partial^2 X^m - 2\partial\psi_m \partial X^m$$
$$L(X,\psi) \equiv L_2 = 2\partial\psi_m\psi^m - \partial X_m \partial X^m$$
$$G = -\frac{1}{2}\psi_m \partial X^m$$

(50)

and

$$F(\varphi,\lambda) = \lambda\partial^2\varphi - 2\partial\lambda\partial\varphi$$
$$L(\varphi,\lambda) = 2\partial\lambda\lambda - (\partial\varphi)^2$$
$$G_L = -\frac{1}{2}\lambda\partial\varphi$$

(51)

where ϕ is the Liouville coordinate (or a coordinate for S^1 compactified direction in the critical $d=10$ case) and λ is superpartner of φ. In uncompactified critical case (which is particularly of interest to us in this work), the only generator in R_2 is $\tilde{L}^{\alpha+}$, while all higher cohomologies are empty. In this case, $\tilde{L}^{\alpha+}(w)$ gives rise to 2 commuting gauge symmetry generators $L_m\alpha+ = \partial^m \tilde{L}^{\alpha+}(w)(m=1,2)$ which makes the expression (46) for the BRST charge remarkably simple, so that it can be written as a worldsheet integral

$$Q_1 = \oint \frac{dz}{2i\pi} \{ce^{\phi}F(X,\psi)P^{(1)}_{\phi-\chi} - \frac{1}{8}e^{2\phi-\chi}(L(X,\psi)P^{(2)}_{2\phi-2\chi-\sigma} + 2GF(X,\psi)) - \partial cc\xi L(X,\psi)\}(z)$$

(52)

The nilpotent charge for the compactified or the noncritical case can be obtained by replacing $F(X,\psi) \to F(X,\psi) + F(\varphi,\lambda)$ and $L(X,\psi) \to L(X,\psi) + L(\varphi,\lambda)$ along with $G = G + G_L$, where $G_L = -\frac{1}{2}\lambda\partial\phi + \frac{q}{2}\partial\lambda$ is the supercurrent in super Liouville theory (q is the Liouville

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background charge, which of course is absent in critical compactified case). The BRST charge Q_1 is the element of superconformal $(\beta - \gamma)$ ghost cohomology H_1; it reflects the gauge symmetries originating from the α-generators of R_2. Analogously, unifying the alpha-generators of R_2 and R_4 and using (46) one can construct the BRST charge Q_{R_2,R_4} related to the guge symmetries originating from the first two $b - c$ cohomologies R_2 and R_4. As it turns out that Q_{R_2,R_4} commutes with Q_1, one can define the nilpotent BRST charge

$$Q_2 = Q_{R_2,R_4} - Q_1 \tag{53}$$

reflecting the gauge symmetries originating from R_4. The obtained nilpotent charge Q_2 turns out to be the element of the superconformal $\beta - \gamma$ ghost cohomology H_2. Next, unifying the alpha-generators of R_2, R_4, R_6 and using (46) one can construct the BRST charge Q_{R_2,R_4,R_6} related to the guge symmetries originating from the first three $b - c$ cohomologies R_2, R_4 and R_6 As it turns out that Q_{R_2,R_4,R_6} commutes with Q_1 and Q_2 one can define the nilpotent BRST charge

$$Q_3 = Q_{R_2,R_4,R_6} - Q_1 - Q_2 \tag{54}$$

reflecting the gauge symmetries originating from R_6. The obtained nilpotent charge Q_2 turns out to be the element of the superconformal $\beta - \gamma$ ghost cohomology H_3. In principle, the construction could be continued to higher values of $n > 3$ to construct the sequence of nilpotent BRST charges Q_n reflecting the gauge symmetries associated with the $b - c$ cohomologies R_{2n} of higher ghst numbers. However, as it was mentioned above, the expressions for the gauge symmetry generators associated with the α-transformations of R_{2n} become increasingly complicated for $n > 3$ and at this stage it seems hard to deduce any manifest expressions for Q_n at $n > 3$. The chain of the BRST charges (46) generally defines the RNS theories (as well as kinetic terms of appropriate string field theories) at various space-time backgrounds. The form of these backgrounds is defined by the geometry of the extra dimensions induced by the underlying *global* α-symmetry generators $\tilde{L}^{\alpha_n I}$. Exploration of properties of these backgrounds (typically of the $AdS_m \times CP_n$ or $AdS_m \times S^n$ type is an interesting problem which is currently under investigation [15]. In the next section we shall return to the pure spinor formalism, demonstrating the map between Q_1 and the deformed pure spinor BRST operator in theory with the nilpotent charge (52) and the leading singularity order of $(z - w)^{-16}$ in OPE $\lambda(z)\lambda(w)$ of two deformed pure spinor variables. In the concluding section we shall attempt to extrapolate our results to relate

the higher order Q_n charges (53), (54), (47) in RNS approach to deformed pure spinors with higher order OPE singularities.

3. Sequence of New BRST Charges in RNS Theory and Deformed Pure Spinors

In this section we discuss the relation between the sequence of BRST charges described in the previous section and deformed pure spinor theories outlined in section 1. The results of this section (as well as the discussion in the concluding section) apply to *critical* ten-dimensional superstring theories (with one of the dimensions possibly compactified on S^1) We already have pointed out that the standard BRST charge Q_0 of the RNS theory can be obtained (up to similarity transformation) from the deformed pure spinor $BRST$ charge with λ's having a double pole OPE singularity, using the RNS representation (5) for λ^α and (9) for d_α. The goal now is to find an interpretation for the alternative BRST charges $Q_n(n = 1, 2, 3, ...)$ in RNS approach in terms of the deformed pure spinors theories with BRST charge having the form (1) but with the different RNS representations of λ (and hence different OPE structures). Namely, we shall look for the RNS representations of λ's which are commuting space-time spinors and dimension zero primary fields, satisfying the relaxed pure spinor condition - that is, the condition (2) fufilled up to terms with vanishing normal ordering with the appropriate OPE terms of $d_\alpha(z)d_\beta(w)$. For that we shall require that λ still can be written BRST commutator: $\lambda^\alpha = \{Q_0, \theta^\alpha\}$ with θ^α still being a dimension 0 primary field and an anticommuting space-time spinor but at higher superconformal ghost numbers (not related to the ghost number $\frac{1}{2}$ expression (6) by any picture-changing transformation). We start with the ghost number $\frac{3}{2}$. The natural way of obtaining a ghost number $\frac{3}{2}$ anticommuting spinor, unrelated to the standard ghost number $\frac{1}{2}$ RNS representation of the Green-Schwarz variable θ is to act on (6) with the truncated α-generator of minimal ghost number 1:

$$\tilde{\theta}^\alpha(z) = [L^{\alpha_1+}, \theta^\alpha] \equiv [\oint \frac{dw}{2i\pi} e^\phi (\psi_m \partial^2 X^m - 2\partial\psi_m \partial X^m)(w), e^{\frac{1}{2}\phi}\Sigma^\alpha(z)]$$
$$= e^{\frac{3}{2}\phi}\tilde{\Sigma}^\beta \gamma^m_{\alpha\beta}(2\partial^2 X_m + \partial X_m \partial\phi)(z) \tag{55}$$

(as previously, $\Sigma, \tilde{\Sigma}$ are the space-time spinors of opposite GSO parities) The $\tilde{\theta}^\alpha$ field is a dimension 0 primary field and a space-time spinor at ghost number $\frac{3}{2}$, *not* related to the ghost number $\frac{1}{2}$ version (6) of the Green-Schwarz variable by picture-changing. Next, one defines the new "pure spinor" variable $\tilde{\lambda}^\alpha$ as the BRST commutator of Q_0 with $\tilde{\theta}^0$:

$$\tilde{\lambda}^\alpha = \{Q_0, \tilde{\theta}^\alpha\} \tag{56}$$

The evaluation of the commutator is quite straightforward giving the answer

$$\tilde{\lambda}^\alpha = \tilde{\lambda}_1^\alpha + \tilde{\lambda}_2^\alpha + \tilde{\lambda}_3^\alpha + \tilde{\lambda}_4^\alpha + \tilde{\lambda}_5^\alpha \tag{57}$$

where

$$
\begin{aligned}
\tilde{\lambda}_1^\alpha = c e^{\frac{3}{2}\phi} \gamma_m^{\alpha\beta} [\partial\tilde{\Sigma}_\beta (2\partial^2 X^m + \partial X^m \partial\phi) \\
+ \tilde{\Sigma}_\beta (2\partial^3 X^m + 4\partial^2 X^m \partial\phi + \partial X^m (\frac{3}{2}(\partial\phi)^2 + \partial^2\phi))]
\end{aligned}
\tag{58}
$$

$$
\begin{aligned}
\tilde{\lambda}_2^\alpha = -\frac{1}{2}\gamma_m^{\alpha\beta} e^{\frac{5}{2}\phi-\chi}\Sigma^\lambda \{ (2\partial^2 X^m + \partial X^m \partial\phi)[(\gamma_n)_{\beta\lambda}(2\partial^2 X^n + \partial X^n P^{(1)\phi-\chi}) \\
+ (\gamma_n\gamma_{pq})_{\beta\lambda}\partial X^n \psi^p \psi^q] \\
+ \partial X^m [(\gamma_n)_{\beta\lambda}(\frac{1}{2}P^{(2)}_{\phi-\chi}\partial X^n + P^{(1)}_{\phi-\chi}\partial^2 X^n \\
+ \frac{1}{2}\partial^3 X^n + \frac{1}{2}\partial X^n (\partial\psi_s \psi^s)) + (\gamma_n\gamma_{pq})_{\beta\lambda}\psi^p \psi^q (2\partial^2 X^n + \partial X^n P^{(1)\phi-\chi})]\}
\end{aligned}
\tag{59}
$$

$$
\begin{aligned}
\tilde{\lambda}_3^\alpha = e^{\frac{5}{2}\phi-\chi}\Sigma^\alpha \{ \frac{5}{6}P^{(4)}_{\phi-\chi} - \frac{2}{3}\partial\phi P^{(3)}_{\phi-\chi} + \partial\psi_s \psi^s (-\frac{5}{8}P^{(2)}_{\phi-\chi} - 2\partial\phi P^{(1)}_{\phi-\chi}) \\
+ \partial^2\psi_s \psi^s P^{(1)}_{4\phi-5\chi} - \frac{5}{4}\partial^3\psi_s - \frac{115}{97}\partial^2\psi_s \partial\psi^s \}
\end{aligned}
\tag{60}
$$

$$
\begin{aligned}
\tilde{\lambda}_4^\alpha = e^{\frac{5}{2}\phi-\chi}\Sigma^\lambda (\gamma_{pq})_{\alpha\lambda} \{ \psi^p \psi^q (\frac{10}{3}P^{(3)}_{\phi-\chi} - 2\partial\phi P^{(2)}_{\phi-\chi}) \\
+ \partial\psi^p \psi^q (10 P^{(2)}_{\phi-\chi} - 4\partial\phi P^{(1)}_{\phi-\chi}) + \frac{5}{3}\partial^2\psi^p \psi^q P^{(1)}_{\phi-\chi} \\
+ 5\partial^3\psi^p \psi^q - \frac{25}{3}\partial^2\psi^p \partial\psi^q - \frac{20}{3}\partial\psi_s \psi^s \partial\psi^p \psi^q \}
\end{aligned}
\tag{61}
$$

$$
\tilde{\lambda}_5^\alpha = -\frac{1}{8}b e^{\frac{7}{2}\phi-2\chi}\gamma_m^{\alpha\beta}\tilde{\Sigma}^\beta (2\partial^2 X^m + \partial X_m \partial\phi) P^{(2)}_{2\phi-2\chi-\sigma} \tag{62}
$$

The next step is the evaluation of the normal ordered product $:\tilde{\lambda}^\alpha d_\alpha:$. The calculation is quite lengthy but very similar to the one performed in [4] for the Q_0 case with the standard λ^α and d_α given by (5), (9). The result is given by

$$
\begin{aligned}
:\tilde{\lambda}^\alpha d_\alpha:(z) = \{ c e^\phi F(X,\psi) P^{(1)}_{\phi-\chi} - \frac{1}{8}e^{2\phi-\chi}(L(X,\psi)P^{(2)}_{2\phi-2\chi-\sigma} + 2GF(X,\psi)) \\
+ \frac{7}{2}\partial c c \xi L(X,\psi)(z) \}
\end{aligned}
\tag{63}
$$

This looks similar to the integrand of the Q_1 BRST charge, except for the last term, having the normalization different from (52). It isn't difficult to see, however, that the worldsheet integral of $:\tilde{\lambda}^\alpha d_\alpha:$ is different from Q_1 just by a similarity transformation. Namely, it isn't difficult to check that

$$
[Q_1, \partial c c \partial^2 \xi \xi e^{-2\phi}] = -\frac{9}{8}\partial c c \xi L(X,\psi) \tag{64}
$$

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Writing

$$\oint \frac{dz}{2i\pi} \tilde{\lambda}^\alpha d_\alpha = Q_1 + \frac{9}{2} \oint \frac{dz}{2i\pi} \partial c c \xi L(X, \psi) \tag{65}$$

it is easy to see that

$$\oint \frac{dz}{2i\pi} \tilde{\lambda}^\alpha d_\alpha = e^{-R_1} Q_1 e^{R_1} \tag{66}$$

with

$$R_1 = -4 \oint \frac{dz}{2i\pi} \partial c c \partial^2 \xi \xi e^{-2\phi} \tag{67}$$

The nilpotence of Q_1 ensures that the deformed pure spinor variable $\tilde{\lambda}^\alpha$ satisfies the "weak" pure spinor condition (that is, up to the terms with the vanishing normal ordering with Π_m). It has a singular OPE with itself, with the leading singularity order equal to 16:

$$\tilde{\lambda}_\alpha(z)\tilde{\lambda}_\beta(w) \sim \frac{\gamma^m_{\alpha\beta} \partial b b e^{7\phi - 4\chi} \psi_m(w)}{(z-w)^{16}} + ... \tag{68}$$

This defines the map of the $BRST$ charge Q_1 (related to the gauge symmetries derived from the R_2 BRST cohomology) to deformed pure spinor theory with the leading OPE singularity order of 16 and presents the main result of this work. In the concluding section we will discuss possible generalizations of this result relating the RNS charges Q_n derived from higher $b-c$ cohomologies R_{2n} and interacting pure spinors with higher order singularities.

5. Discussion and Conclusion

In this paper we particularly have constructed the map between one of the recently derived BRST charges [9] (related to local ghost matter mixing gauge symmetries in RNS formalism) and the BRST charge (66) with the $\tilde{\lambda}$ variables subject to relaxed pure spinor constraint and with singular operator products. The $\tilde{\lambda}$ variables have been obtained as BRST anticommutator of the standard BRST charge Q_0 with the Green Schwarz variables transformed by the truncated α-symmetry generators of ghost number 1. The natural question is whether the construction can be generalized to relate the higher order BRST charges Q_n to deformed pure spinors with higher order OPE singularities. The natural way to extend the construction demonstrated in this paper is to start with the transformations of the Green-Schwarz variables θ_α with truncated α-generators of minimal ghost number n:

$$\theta_\alpha \to \theta_\alpha^{\underline{N}} = [L^{\underline{N}}, \theta_\alpha] \tag{69}$$

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where $\underline{N} = \alpha_n I; I \equiv (\alpha_{n-1}, ..., \alpha_1)$ labels the indices of the truncated α-symmetry generators (obviously excluding the generators that are the Lorenz vectors) at minimal ghost number n. Here and below we shall use the underlined index notation for the generators in order to avoid any confusion with space-time fermionic index α. Next, one constructs the commuting spinor variables (primary field of conformal dimension zero) as

$$\lambda_{\alpha}^{\underline{N}} = \{Q_0, \theta_{\alpha}^{\underline{N}}\}. \tag{70}$$

Then, for each ghost number n one has to find certain linear combination of the commuting spinor variables $\lambda_{\alpha}^{(n)} = \sum_{I(n)} \lambda_{\alpha}^{\underline{N}(n)}$ where $\underline{N}(n) \equiv (\alpha_n I(n))$ with $I(n) = (\alpha_{n-1}...\alpha_n)$ so the summation is over the $n-1$ different values of the index $I(n)$. The linear combination has to be constructed so that the resulting $\lambda_{\alpha}^{(n)}$ field satisfies the weak pure spinor constraint (i.e. all the singular and the normal ordered terms of the OPE $\lambda_{\alpha}^{(n)}(z)\lambda_{\beta}^{(n)}(z)$ have vanishing normal ordering with the relevant terms of the OPE $d^{\alpha}(z)d^{\beta}(w)$. If the conjectured correspondence between the deformed pure spinor theories and the RNS models defined by the Q_n BRST charges is correct for $n > 1$, such a combination should exist and should be unique for each n. Finally, one constructs the charge

$$Q_n^{PS} = \oint \frac{dz}{2i\pi} \lambda_{\alpha}^{(n)} d^{\alpha} \tag{71}$$

by computing the normal ordered product of $\lambda_{\alpha}^{(n)}$ with d^{α} of (9). The constructed charges, written in terms of the RNS variables should presumably reproduce the Q_n charges in the RNS formalism related to the R_{2n} cohomologies; however, due to the complexity of all the relevant expressions beyond $n = 1$ we haven't been able so far to prove such a correspondence for higher values of n. The leading term in the OPE of two $\lambda_{(n)}$'s is given by

$$\lambda_{\alpha}^{(n)}(z)\lambda_{\beta}^{(n)}(w) \sim \frac{\gamma_{\alpha\beta}^m \partial b b e^{(2n+5)\phi-4\chi}\psi_m(w)}{(z-w)^{-2(n^2+6n+1)}}, \tag{72}$$

so the correspondence between deformed pure spinors and R_{2n} related BRST charges in RNS formalism (if it holds) implies that orders of R_{2n} cohomologies giving rise to Q_n BRST charges in RNS formalism, should be in one to one correspondence with the leading singularity orders of the OPE of deformed pure spinors, given by $2n^2 + 12n + 2$.

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